On the Capacity and Simulation of 3-D MIMO Mobile-to-Mobile Relay Fading Channels

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Abstract—This paper proposes a theoretical framework for the evaluation of the capacity of multiple-input multiple-output (MIMO) mobile-to-mobile (M-to-M) fading channels in dual-hop amplify-and-forward (AF) single-relay wireless communications networks. The evaluation is based on a recently introduced three-dimensional (3-D) geometry-based reference model for these channels. From this model, 3-D deterministic and statistical sum-of-sinusoids (SoS) based simulation models are also developed, under 3-D non-isotropic scattering conditions. The performance of the simulation models is verified through simulation results.

Keywords—Amplify-and-forward relay links; channel capacity; MIMO channels; mobile-to-mobile channels; simulation; 3-D scattering

I. INTRODUCTION

Mobile-to-Mobile (M-to-M) communications intend to substantially contribute to mobile ad-hoc networks, intelligent transportation systems, broadband mobile multimedia services, and relay-based cellular networks by expanding the network range and increasing the mobility support. However, harsh multipath fading effects usually degrade the transmission link quality. Combining the features of cooperative diversity via wireless relays with the benefits of multiple-input multiple-output (MIMO) technology makes it possible to fulfill the growing demands of enhanced reliability and flexibility with increased channel capacity. However, the analysis, design, and performance evaluation of MIMO M-to-M relay-based systems necessitate a thorough investigation of the underlying radio channel. Therefore, realistic channel models are indispensable.

In [1], [2] two-dimensional (2-D) two-ring models for MIMO M-to-M channels were proposed, while a three-dimensional (3-D) two-cylinder model was presented in [3]. Nevertheless, the statistical properties of these channels are quite different from MIMO M-to-M relay channels, where a transceiver unit called a relay (R) is interposed between a source (S) and a destination (D). In this case, all the mobiles stations are affected by the scatterers in their local vicinity. Based on the two-ring models, a three-ring model for MIMO M-to-M relay fading channels was proposed in [4], where a separate ring of scatterers around the relay is considered in addition to the rings around the source and the destination. This model assumes that the radio waves travel only in the horizontal plane. However, in densely built-up urban areas, the antenna arrays are usually located lower than the surrounding scatterers and the scattered waves propagate by diffraction from vertical structures, i.e., buildings, to the street level. Then, the waves may travel in both horizontal and vertical planes, i.e., 3-D scattering occurs. Thus, azimuth and elevation angles for the incidence of the radio waves should be introduced [3].

Recently, the authors addressed the design of a 3-D reference geometry-based model for MIMO M-to-M fading channels in dual-hop amplify-and-forward (AF) single-relay networks [5]. This model assumes that the scatterers in the vicinity of the source, the relay, and the destination lie on the surface of three cylinders, which reflect the influence of three heterogeneous 3-D non-isotropic scattering environments. In this paper, using this reference model, the channel capacity of uniform linear antenna arrays (ULAs) is studied and evaluated for different model parameters. The reference model is practically non-realizable, since an infinite number of local scatterers is assumed. By adopting the sum-of-sinusoids (SoS) principle [6] due to its reasonably low computational costs and the explicit inclusion of spatial information, e.g., the multipath angles of arrival/departure, this paper proposes deterministic and statistical simulation models with a finite number of scatterers, under the framework of the reference model.

The remainder of the paper is organized as follows. Section II presents the system model and defines the channel capacity. Section III details the system geometry, while Section IV describes the 3-D reference model. Deterministic and statistical simulation models are developed in Section V. Section VI provides numerical and simulation results. Finally, conclusions are drawn in Section VII.

II. SYSTEM MODEL AND CHANNEL CAPACITY

This paper considers a narrowband MIMO M-to-M relay-based communications system with $L_S$, $L_R$, and $L_D$ antenna elements at the source, the relay, and the destination, respectively. All antennas are omni-directional with low height and are numbered as $1 \leq p \leq q \leq L_S$, $1 \leq q \leq L_R$, and $1 \leq u \leq L_D$, respectively. It is assumed that the direct link between source and destination is obstructed. Then, the entire S-R-D MIMO system can be separated into the S-R and R-D MIMO subsystems.

The received signal vector at the relay is given by

$$y_R(t) = \frac{P_S}{L_S} H_{sr}(t) s(t) + n_R(t), \quad (1)$$

where $H_{sr}(t)$ is the channel transfer function between the source and the relay, $s(t)$ is the transmitted signal, and $n_R(t)$ is the additive white Gaussian noise.
where \( P_S \) is the total source transmit power, \( \mathbf{H}_{sr}(t) \in \mathbb{C}^{L_S \times L_T} \) is the channel transfer matrix for the transmission link from the source to the relay, \( \mathbf{s}(t) \in \mathbb{C}^{L_T \times 1} \) is the signal vector transmitted by the source with \( \mathbb{E} \left[ \| \mathbf{s} \|^2 \right] = \mathbf{I}_{L_T} \), and \( \mathbf{n}_R(t) \sim \mathcal{CN}(\mathbf{0}, N_{o,R} \mathbf{I}_{L_R}) \) is the \( L_R \times 1 \) zero-mean circularly symmetric complex Gaussian noise vector at the relay, where \( N_{o,R} \) is the noise power spectral density (PSD). The relay normalizes the received signal power to unity, which is \( \mathbb{E} \left[ \| \mathbf{n}_R \|^2 \right] = (P_S + N_{o,R}) L_R \), and retransmits it to the destination with mean total power \( P_R \). The received signal vector at the destination is

\[
y_D(t) = \left( P_R \over (P_S + N_{o,R}) L_R \right) \mathbf{H}_{rd}(t) \mathbf{y}_R(t) + \mathbf{n}_D(t), \tag{2}
\]

where \( \mathbf{H}_{rd}(t) \in \mathbb{C}^{L_D \times L_T} \) is the channel transfer matrix for the transmit link from the relay to the destination.\( \mathbf{n}_D(t) \sim \mathcal{CN}(\mathbf{0}, N_{o,D} \mathbf{I}_{L_D}) \) is the noise vector at the destination and \( N_{o,D} \) is the noise PSD. Using (1) and (2), the input-output relation for the overall S-R-D system becomes

\[
y_D(t) = \left( P_R \over (P_S + N_{o,R}) L_R \right) \mathbf{H}(t) \mathbf{s}(t) + \mathbf{n}_D(t), \tag{3}
\]

where \( \mathbf{H}(t) = \mathbf{H}_{rd}(t) \mathbf{H}_{sr}(t) \in \mathbb{C}^{L_D \times L_T} \) is the effective channel transfer matrix describing the MIMO relay fading channel and \( \mathbf{n}_D(t) = \sqrt{P_R / \left( (P_S + N_{o,R}) L_R \right)} \mathbf{H}_{rd}(t) \mathbf{n}_R(t) + \mathbf{n}_D(t) \) is the effective noise of the S-R-D system. This channel is assumed static for two consecutive timeslots, i.e., the period needed to complete the AF operation.

Conditioning on \( \mathbf{H}_{sr} \) and \( \mathbf{H}_{rd} \), the effective noise of the system is Gaussian distributed with covariance matrix

\[
\mathbf{R}_{n_D}(t) = N_{o,D} \left[ \mathbf{I}_{L_D} + \rho_D \mathbf{I}_{L_R} \mathbf{H}_{rd}(t) \mathbf{H}_{sr}^H(t) \right], \tag{4}
\]

where \( \mathbf{I}_{L_D} \) is an identity matrix of size \( L_D \), and \( \rho_D = P_S / N_{o,R} \) and \( \rho_D = P_R / N_{o,D} \) denote the mean received signal-to-noise ratio (SNR) per antenna at the relay and destination node, respectively. By assuming that the channel is known to the destination, multiplying \( \mathbf{y}_D \) on the left by \( \mathbf{R}_n^{-1/2}(t) \), applying the matrix inversion lemma to calculate \( \mathbf{R}_n^{-1}(t) \), and using known results from information theory, we obtain the instantaneous channel capacity (in bits per second per hertz):

\[
C(t) = \log_2 \det \left[ \mathbf{I}_{L_D} + \mathbf{Z} \mathbf{H}(t) \right] \times \left[ \mathbf{I}_{L_D} - \mathbf{H}_{rd}(t) \mathbf{G}^{-1}(t) \mathbf{H}_{rd}^H(t) \mathbf{H}(t) \right], \tag{5}
\]

where

\[
\mathbf{X} = {\frac{\rho_S \rho_D}{L_S L_R (1 + \rho_D)}} \tag{6}
\]

\[
\mathbf{G}(t) = \mathbf{H}_{rd}^H(t) \mathbf{H}_{rd}(t) + {\frac{1 + \rho_R}{\rho_D}} L_S \mathbf{I}_{L_D}. \tag{7}
\]

The elements of \( \mathbf{H}(t) \) can be directly generated using the proposed simulation models in Section V. Alternatively, since \( \mathbf{H}(t) = \mathbf{H}_{rd}(t) \mathbf{H}_{sr}(t) \), \( \mathbf{H}(t) \) can be indirectly derived using the reference model in Section IV. In particular, \( \mathbf{H}_{sr}(t) \) can be derived as follows \[7\]

\[
\mathbf{H}_{sr} = \mathbf{R}_{n_D}^{1/2} \mathbf{h}_R \mathbf{R}_{s}^{-1/2}, \tag{8}
\]

where \( \mathbf{h}_R \) is a \( L_R \times L_S \) stochastic matrix with independent and identically distributed (i.i.d.) zero-mean complex Gaussian entries, \( \mathbf{R}_s \) and \( \mathbf{R}_{sr} \) are the \( L_S \times L_S \) source and \( L_R \times L_R \) relay (S-R subsystem) correlation matrices, \( (\cdot)^{1/2} \) denotes the matrix square root operation, and \( (\cdot)^T \) denotes the transpose operation. The MIMO channel matrix \( \mathbf{H}_{rd} \) can be generated by following a similar procedure to that used to generate \( \mathbf{H}_{sr} \).

III. THE 3-D THREE-CYLINDER SCATTERING MODEL

Fig. 1 shows the 3-D scattering model for a 2x2x2 MIMO M-to-M relay fading channel \[5\]. One observes that the relay is positioned at an angle \( \omega_S \) with respect to the source, while the location of the relay seen from the destination is described by the angle \( \omega_D \). The antenna spacing at the source, relay, and destination is denoted by \( \delta_S \), \( \delta_R \), and \( \delta_D \), respectively, while \( \Omega_S \), \( \Omega_R \), and \( \Omega_D \) are the corresponding antenna array centers. Moreover, the angles \( \theta_S \), \( \theta_R \), and \( \theta_D \) represent the orientation of each antenna array, relative to the x-axis, and \( \psi_S \), \( \psi_R \), and \( \psi_D \) describe the elevation angle of the elements \( p \), \( q \), and \( u \), respectively, relative to the x-y plane. In addition, the source, relay, and destination are moving with speeds \( v_S \), \( v_R \), and \( v_D \), respectively, while the angles \( \gamma_S \), \( \gamma_R \), and \( \gamma_D \) control the moving directions. Note that \( \hat{A} \) denotes the projection of a point \( A \) onto the x-y plane.

It is assumed that \( M \rightarrow \infty \) scatterers denoted by \( \mathbf{s}_R^{(m)} \) \( (m = 1, 2, ..., M) \) are situated around the source, on the surface of a cylinder of radius \( R_S \). Similarly, \( \mathbf{s}_R^{(k)} \) \( (k = 1, 2, ..., K) \) and \( \mathbf{s}_D^{(l)} \) \( (l = 1, 2, ..., L) \) scatterers in the vicinity of the relay with \( K = L \rightarrow \infty \) and \( \mathbf{s}_D^{(k)} = \mathbf{s}_R^{(k)} \) for \( k = l \) lie on a surface of a cylinder of radius \( R_D \), while \( N \rightarrow \infty \) scatterers denoted by \( \mathbf{s}_D^{(n)} \) \( (n = 1, 2, ..., N) \) are situated ad the destination, on the surface of a cylinder of radius \( R_D \). Only double-bounce non-line-of-sight (NLoS) propagation conditions are considered, which are dominant in urban
denote the azimuth angle of departure (AAoD) and the azimuth angle of arrival (AAoA), respectively, of the wave transmitted from the source and impinged on $S_{(m)}$. While $a_{sr}^{(i)}$ and $\beta_{sr}^{(i)}$ are the azimuth angle of arrival (AAoA) and the elevation angle of arrival (EAAoA), respectively, of the wave scattered from $S_{(i)}$ and received at the relay. Besides, $a_{rd}^{(i)}$ and $\beta_{rd}^{(i)}$ denote the AAoA and the EAAoA, respectively, of the wave scattered from $S_{(i)}$ and received at the destination. Due to the heterogeneity of the scattering environments and the double-bounce scattering, the angles of departure are independent from the angles of arrival, while the azimuth and elevation angles are also independent [9].

It is assumed that the inequalities $\max \{R_s, R_r\} \ll d_{sr}$ and $\{R_r, R_d\} \ll d_{rd}$ hold, where $d_{sr}$ and $d_{rd}$ denote the distances $\hat{O}_s$ to $\hat{O}_r$ and $\hat{O}_r$ to $\hat{O}_d$, respectively. It is also assumed that $\max \{\delta_s, \delta_r, \delta_d\} \ll \min \{R_s, R_r, R_d\}$. Finally, it is assumed that $d_{sr} \leq 4R_sR_rL_s / (\lambda (L_s-1)(L_r-1))$ and $d_{rd} \leq 4R_rR_dL_d / (\lambda (L_r-1)(L_d-1))$, where $\lambda$ is the carrier wavelength. Hence, the channel does not experience keyhole behavior [7].

IV. THE 3-D REFERENCE MODEL

The reference model assumes that the number of scatterers in the vicinity of the source, the relay, and the destination is infinite. Then, for $L_r = 2$, the received complex faded envelope of the link from the source element $p$ to the destination element $q$ via the relay element $r$ is given by [5]

$$h_{pu} (t) = \lim_{K \to \infty} \frac{1}{\sqrt{KLMN}} \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{n=1}^{N} A_s A_d C_s C_r$$

$$\times \left( B_{SR} B_{RD}^* \right) \exp \left[ j \varphi_{mk} + j \varphi_{pm} + j \varphi_{sr} + j \varphi_{rd} \right]$$

$$\times \exp \left[ j 2 \pi t (f_s + f_d + f_{SR} + f_{RD}) \right],$$

where

$$A_s = \exp \left[ j \pi (3 - 2 p) \delta_s \sin \psi_s \sin \beta_s^{(m)} / \lambda \right]$$

$$\times \exp \left[ j \pi (3 - 2 p) \delta_s \cos \psi_s \cos \beta_s^{(m)} \cos \theta_s - d_s^{(m)} / \lambda \right],$$

$$A_d = \exp \left[ j \pi (3 - 2 n) \delta_r \sin \psi_r \sin \beta_r^{(n)} / \lambda \right]$$

$$\times \exp \left[ j \pi (3 - 2 n) \delta_r \cos \psi_r \cos \beta_r^{(n)} \cos \theta_r - d_r^{(n)} / \lambda \right],$$

$$B_{SR} = \exp \left[ j \pi (3 - 2 q) \delta_s \sin \psi_r \sin \beta_{sr}^{(i)} / \lambda \right]$$

$$\times \exp \left[ j \pi (3 - 2 q) \delta_s \cos \psi_r \cos \beta_{sr}^{(i)} \cos \theta_r - d_{sr}^{(i)} / \lambda \right],$$

$$B_{RD} = \exp \left[ j \pi (3 - 2 q) \delta_r \sin \psi_r \sin \beta_{rd}^{(i)} / \lambda \right]$$

$$\times \exp \left[ j \pi (3 - 2 q) \delta_r \cos \psi_r \cos \beta_{rd}^{(i)} \cos \theta_r - d_{rd}^{(i)} / \lambda \right],$$

$$C_{SR} = \exp \left[ j 2 \pi R_s \cos \left( d_s^{(m)} - \omega_s \right) - R_r \cos \left( d_{sr}^{(i)} - \omega_r \right) \right] / \lambda,$$

$$C_{RD} = \exp \left[ j 2 \pi R_r \cos \left( d_r^{(n)} - \omega_r \right) - R_d \cos \left( d_{rd}^{(i)} - \omega_d \right) \right] / \lambda,$$

$$\varphi_{sr} = -2 \pi (d_{sr} + R_s + R_r) / \lambda,$$

$$\varphi_{rd} = -2 \pi (d_{rd} + R_r + R_d) / \lambda,$$

$$f_s = f_{s,max} \left( d_s^{(m)} - \gamma_s \right) \cos \beta_s^{(m)},$$

$$f_d = f_{d,max} \left( d_r^{(n)} - \gamma_d \right) \cos \beta_r^{(n)},$$

$$f_{SR} = f_{SR,max} \left( d_{sr}^{(i)} - \gamma_r \right) \cos \beta_{sr}^{(i)},$$

$$f_{RD} = f_{RD,max} \left( d_{rd}^{(i)} - \gamma_r \right) \cos \beta_{rd}^{(i)},$$

and $f_{s,max} = v_s / \lambda$, $f_{d,max} = v_d / \lambda$, and $f_{SR,max} = v_{sr} / \lambda$ are the maximum Doppler shifts associated with the source, destination, and relay, respectively. The phases $\varphi_{mk}$ and $\varphi_{kn}$ are random variables uniformly distributed in the interval $[0,2\pi]$.

The space-time correlation function (STCF) between $h_{pu} (t)$ and $h_{pu'} (t)$ is given by

$$R_{pu,pu'} (\delta_s, \delta_r, \delta_d, \tau) = E_R \left[ h_{pu} (t) h_{pu'}^* (t + \tau) \right],$$

where

![Figure 1. The three-cylinder scattering model for a 2x2x2 MIMO M-to-M single-relay dual-hop fading channel [5].](image-url)
where $(\cdot)^*$ denotes complex conjugate operation and $E[\cdot]$ is the statistical expectation operator. Since the number of scatterers is infinite, the discrete AAoD, AAoA, EAOd, and EAOA can be replaced with continuous random variables $\theta_S$, $\theta_D$, $\phi_S$, $\phi_D$, $\beta_S$, $\beta_D$, $\beta_{GR}$, $\beta_{GD}$, $\beta_{RD}$, $\alpha$, and $\beta_D$, respectively, with a joint probability density function (pdf). This joint pdf can be decomposed to a product of eight separate pdfs, since the aforementioned random variables are mutually independent.

The von Mises pdf [10] is used to characterize the EAoDs and AAoAs. Considering the AAoD $\theta_S$, this pdf is defined as

$$f(\theta_S) = \frac{\pi}{4|\beta_{SS}|} \cos \left(\frac{\pi \theta_S}{2 \beta_{SS}}\right), \quad |\theta_S| < \frac{\pi}{2},$$

(13)

where $\beta_{SS}$ is the maximum value of $\beta_S$, usually less than 20°, value typical for M-to-M communications [9].

Using (9), (12), and (13) and the results in [2, 4, and 5], (11) can be written as

$$R_{\mu,\nu}(\delta_s, \delta_d, \tau) = R_S(\delta_s, \tau) R_D(\delta_d, \tau),$$

(14)

where $R_S(\delta_s, \tau)$ and $R_D(\delta_d, \tau)$ denote the source and the destination STCFs, respectively,

$$R_S(\delta_s, \tau) = \sum_{\delta_d} R_{SR}(\delta_s, \tau) R_{SR}(\delta_d, \tau),$$

(15)

is the relay STCF of the S-R-D system, and $R_{SR}(\delta_s, \tau)$ and $R_{RD}(\delta_d, \tau)$ denote the relay STCF of the S-R and R-D systems, respectively. The source STCF is derived as [3, 5]

$$R_S(\delta_s, \tau) = \frac{\pi}{|\beta_{SS}|} \cos \left(\frac{\pi \delta_s}{2 \beta_{SS}}\right) I_0 \left(\frac{\beta_{SS}}{2 |\beta_{SS}|} \delta_s \tau \cos \beta_S \right) \times \exp \left(j2\pi(\nu - \nu_S) \delta_s \sin \psi_S \sin \beta_S / \lambda \right) d\delta_s,$$

(16)

where

$$\nu_S = j2\pi(\nu - \nu_S) \delta_s \cos \theta_S \cos \psi_S / \lambda - j2\pi \nu_{max} \cos \psi_S + k_S \cos \mu_S / \cos \beta_S,$$

(17)

$$\gamma_S = j2\pi(\nu - \nu_S) \delta_s \sin \theta_S \cos \psi_S / \lambda - j2\pi \nu_{max} \sin \psi_S + k_S \sin \mu_S / \cos \beta_S.$$

(18)

The other STCFs can be similarly defined from (16)-(18) by replacing the indices. Clearly, the STCF in (14) is accurate only over a time duration that is much smaller than $\min\{R_S / \nu_S, R_D / \nu_D\}$.

V. 3-D SIMULATION MODELS

The reference model assumes an infinite number of scatterers and prevents practical realization. However, from this model, SoS-based simulation models with a finite number of scatterers can be developed. Using (9) and the results in [3], the following received complex faded envelope is proposed:

$$h_{pu}(t) = (K_p K_s L_s L_r M_s M_r N_s N_r)$$

$$\times \sum_{k_s} \sum_{k_r} \sum_{m_S} \sum_{m_D} A_{S,m_S} A_{D,m_D}$$

$$\times C_{SR,k_s,k_r} C_{RD,k_r,k_d} B_{SR,k_s,k_r} B_{RD,k_r,k_d} + B_{SR,k_s,k_d} B_{RD,k_r,k_d}$$

$$\times \exp \left(j \phi_{m_S,m_D,k_s,k_d} + j \phi_{S,k_s,k_d} + j \phi_{D,k_r,k_d} \right),$$

(19)

where $K_p K_s = K$, $L_s L_r = L$, $M_s M_r = M$, $N_s N_r = N$, $A_{S,m_S} A_{D,m_D} = \exp \left[j \pi \delta_S \sin \psi_S \sin \beta_S / \lambda \right]$.

$$\times \exp \left[j \pi \delta_D \sin \psi_D \sin \beta_D / \lambda \right],$$

$$A_{S,m_S} A_{D,m_D} = \exp \left[j \pi \delta_S \sin \psi_S \sin \beta_S / \lambda \right],$$

$$\times \exp \left[j \pi \delta_D \sin \psi_D \sin \beta_D / \lambda \right],$$

$$B_{SR,k_s,k_r} = \exp \left[j \pi \delta_S \sin \psi_S \sin \beta_S / \lambda \right],$$

$$\times \exp \left[j \pi \delta_D \sin \psi_D \sin \beta_D / \lambda \right],$$

$$B_{RD,k_r,k_d} = \exp \left[j \pi \delta_D \sin \psi_D \sin \beta_D / \lambda \right].$$

$$C_{SR,k_s,k_r} = \exp \left[j 2\pi R_S \cos \left(\nu_S - \theta_S \right) \right],$$

$$- R_S \cos \left(\nu_S - \theta_S \right),$$
Note that the indices A and E in (19) and (20) are associated with the multipath azimuth and elevation angles, respectively.

Using (11), (19), and (20), the STCF between \( h_{p_p}(t) \) and \( h_{p_p}'(t) \) is derived as follows

\[
R_{p_p,p_p}'(\delta_1, \delta_2, \delta_3, \tau) = \frac{1}{M_A M_E} \sum_{m_A, m_E} A_{m_A, m_E} \exp \left[ -j 2\pi \frac{\sin \left( \frac{\pi}{2} \left( \delta_1 + \delta_2 - \delta_3 - \delta_4 \right) \right)}{M_A M_E} \right]
\]

To successfully approximate the statistical properties of the reference model, the multipath azimuth and elevation angles should be estimated. Although the LPNM [2] provides maximum accuracy and versatility, it requires the minimization of multiple error functions in order to obtain proper correlation properties. This minimization must be simultaneously applied for the azimuth and elevation angles. Thus, simpler and less complex methods are desirable.

First, a deterministic (ergodic statistical) model is developed. This model computes fixed values for the azimuth and elevation angles. One can similarly generate the other azimuth and elevation angles. The complexity of this model is reduced by [12]. One can similarly generate the other azimuth and elevation angles. The complexity of this model is reduced by [12].

A statistical (Monte Carlo) model is also developed. The statistical properties of this model vary for each simulation trial and converge to the theoretical ones after averaging over a sufficient number of simulation trials for an arbitrary finite number of scatterers. Based on [3], we generate the AAOd \( a_{AA}^{(ae)} \) and the EAOd \( \beta_{EE}^{(ae)} \) as follows

\[
a_{AA}^{(ae)} = F^{-1} \left( \left( m_e - 0.5 \right) / M_E \right),
\]

\[
\beta_{EE}^{(ae)} = \frac{2 \beta}{\pi} \arcsin \left( \frac{2 m_e - 1}{M_E} \right)
\]

for \( m_e = 1, \ldots, M_E \). The function \( F^{-1}() \) denotes the inverse function of the von Mises cumulative distribution function (cdf) and can be numerically evaluated [12]. One can similarly generate the other azimuth and elevation angles. The complexity of this model is reduced by choosing a minimum, but also adequate number of scatterers.

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A statistical (Monte Carlo) model is also developed. The statistical properties of this model vary for each simulation trial and converge to the theoretical ones after averaging over a sufficient number of simulation trials for an arbitrary finite number of scatterers. Based on [3], we generate the AAOd \( a_{AA}^{(ae)} \) and the EAOd \( \beta_{EE}^{(ae)} \) as follows

\[
a_{AA}^{(ae)} = F^{-1} \left( \left( m_e - 0.5 \right) / M_E \right),
\]

\[
\beta_{EE}^{(ae)} = \frac{2 \beta}{\pi} \arcsin \left( \frac{2 m_e - 1}{M_E} \right)
\]

for \( m_e = 1, \ldots, M_E \). The function \( F^{-1}() \) denotes the inverse function of the von Mises cumulative distribution function (cdf) and can be numerically evaluated [12]. One can similarly generate the other azimuth and elevation angles. The complexity of this model is reduced by choosing a minimum, but also adequate number of scatterers.
of the deterministic model is obtained for $K_L = M_A = N_A = 30$ and $K_E = M_E = N_E = 4$. Besides, the STCF of the statistical model is obtained for $K_L = M_A = N_A = 20$, $K_E = M_E = N_E = 3$, and 30 simulation trials. One observes that the STCF of the deterministic model closely matches the reference one, whereas the STCF of the statistical model perfectly reproduces the reference one.

VII. CONCLUSIONS

In this paper, the capacity of outdoor MIMO M-to-M fading channels in AF relay-based networks has been studied, under 3-D non-isotropic scattering conditions. Deterministic and statistical simulation models for these channels have been also developed and the inherent advantages of the SoS principle have been exploited. The results have confirmed that the simulation models approximate the statistical properties of the reference model with high precision. To further validate the utility of the reference model, future work could be devoted to comparing its statistical properties with measurement data.

ACKNOWLEDGMENT

This work is co-funded by the European Union and national resources under the National Strategic Reference Framework (NSRF) and the THALES research project: INTENTION.

REFERENCES


![Figure 4](ExampleImage1.png)

**Figure 4.** Ergodic capacity of a 2x2x2 MIMO M-to-M relay channel for different maximum EAoDs and EAoAs and vertically placed antennas.

![Figure 5](ExampleImage2.png)

**Figure 5.** Real part of the STCFs of the reference and simulation models.

Fig. 4 justifies the inclusion of the third dimension of the model and shows the $C_{etz}$ as a function of the maximum EAoDs and EAoAs for $\psi_A = \psi_B = \psi_D = \pi / 2$. One observes that increasing $\beta_{SRm}$ and $\beta_{RDm}$ from 1° to 20°, decreases the capacity by 0.5 bps/Hz. However, increasing $\beta_{SR}$ and $\beta_{RD}$, decreases the capacity loss. Therefore, the 2-D model in [4] overestimates the available capacity.

Fig. 5 examines the performance of the simulation models and compares the real part of their STCF with the real part of the STCF of the reference model. The values of the model parameters used are $L_S = L_R = L_D = 2$, $\delta_S = \delta_D = \delta_R = \lambda$, $\delta_A = \delta_B = \pi / 6$, $\psi_S = \psi_A = \psi_E = \pi / 4$, $k_S = 8$, $k_{SR} = k_{RD} = 5$, $k_D = 2$, $\mu_S = 0^\circ$, $\mu_{SR} = \mu_{RD} = \pi / 6$, $\mu_D = \pi / 3$, $\beta_{SA} = \beta_{SD} = \beta_{DA} = 15^\circ$, $\gamma_S = 0^\circ$, $\gamma_A = \pi / 4$, and $\gamma_D = \pi / 2$. A normalized sampling period $f_{S,max} T_S = 0.01$ is used, where $T_S$ is the sampling period and $f_{S,max} = f_{R,max} = f_{D,max}$. The STCF